

# An Approximate Solution for the Flow Far Downstream of Condensation in a Hypersonic Nozzle

P. M. SHERMAN\* AND D. D. McBRIDE†

The University of Michigan, Ann Arbor, Mich.

THE expansion of a mixture of a vapor and a "carrier" gas in a supersonic or hypersonic nozzle can be assumed to be isentropic. Since the saturation line for most vapors is such that the saturation temperature decrease is less than that of an isentropic expansion for the same decrease in pressure, saturation of the vapor is reached, and subsequent condensation will usually occur. If condensation occurs in the supersonic region, then in the hypersonic region where very little of the vapor is still uncondensed, the mixture will be near the state for an expansion in thermal equilibrium. A summary of the work done in this area is given in Ref. 1.

The purpose of this Note is to show that for the region far downstream of condensation in a nozzle, a simplified solution can be obtained. This solution is adequate and convenient for estimating downstream effects of condensation in wind tunnels and in nozzles for some of the newer propulsion schemes such as colloidal thrusters and EHD generators.

If in addition to the usual assumptions made for two component, equilibrium, one-dimensional, steady-state flow equations, a constant latent heat is assumed, it has been shown that an integrated set of equations can be obtained.<sup>1,2</sup> The integrated equations for the condensing flow are:

$$\rho u \bar{A} = \dot{m} \quad (1)$$

$$p = [q + (1 - K)\alpha] \rho R T / \mu_v \quad (2)$$

$$u = [2C(T_c - T) + 2(K - q)L]^{1/2} \quad (3)$$

$$q = -L/\theta + 1/\theta \{L^2 + 2\theta[C(T_c - T) + KL]\}^{1/2} \quad (4)$$

$$\theta \equiv \left[ \frac{\dot{m} R T}{\mu_v \bar{A} p_v} \right]^2 = \left[ \frac{\dot{m} R T}{\mu_v \bar{A}} \right]^2 \exp \left[ \frac{2L\mu_v}{RT} - 2B \right] \quad (5)$$

$$F(T) \equiv \frac{C}{L} T \ln \frac{T}{T_1} + \frac{(1 - K)RT}{\mu_a L} \ln \frac{q}{K} + q + (1 - K)\alpha - \frac{T}{T_1} [K + (1 - K)\alpha] = 0 \quad (6)$$

where:  $\rho$  is density,  $u$  is velocity,  $\dot{m}$  is mass flow rate per unit area of throat,  $\bar{A}$  is area ratio,  $p$  is pressure,  $q$  is mass fraction of uncondensed vapor,  $K$  is mass fraction of vapor in stagnation chamber,  $\alpha$  is ratio of vapor molecular weight to carrier gas molecular weight,  $\mu_v$  is molecular weight of the vapor,  $C$  is constant pressure specific heat of the vapor-gas mixture in the stagnation chamber,  $T_c$  is temperature in the stagnation chamber,  $T$  is local temperature,  $L$  is latent heat,  $p_v$  is partial pressure of the vapor,  $B$  is a constant in the Clausius-Clapeyron equation for the saturation curve of the vapor,  $T_1$  is the saturation point temperature.

Far downstream where the amount of vapor left uncondensed is very small or where

$$q \ll (1 - K)\alpha \quad (7)$$

and

$$L \ll \{2\theta[C(T_c - T) + KL]\}^{1/2}$$

Received May 22, 1969; revision received July 30, 1969. This research was supported in part by the Aerospace Research Labs, Office of Aerospace Research, the U.S. Air Force, under contract AF 33(615)-67C-1197.

\* Associate Professor, Aerospace Engineering.

† Graduate Student; now Staff Member, Aerothermodynamics Projects Dept., Sandia Corp., Albuquerque, N.M.

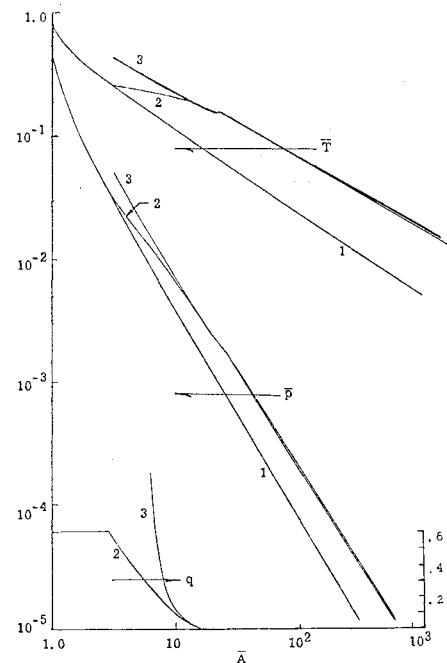


Fig. 1 Comparison of solutions: 1—frozen flow (no condensation); 2—exact computer solution; 3—approximate solution, Eq. (13); ( $\bar{T} \equiv T/T_c$ ,  $\bar{p} \equiv p/p_c$ ).

which means: (8)

$$q \ll [K + C(T_c - T)/L]$$

Eq. (4) simplifies to

$$q = \{(2/\theta)[C(T_c - T) + KL]\}^{1/2} \quad (9)$$

and Eq. (6) becomes

$$F(T) = \Gamma \ln \frac{T}{T_1} + \ln \left\{ \frac{\mu_v}{\dot{m} R K} [2KL + 2C(T_c - T)]^{1/2} \frac{\bar{A}}{T} \right\} + B - \frac{L\mu_v}{RT_1} \left[ 1 + \frac{K}{(1 - K)\alpha} \right] = 0 \quad (10)$$

where

$$\Gamma \equiv \frac{C\mu_v}{(1 - K)\alpha R} = \frac{C\mu_a}{(1 - K)R} \quad (11)$$

If static temperature has decreased so that

$$T \ll (T_c + KL/C) \quad (12)$$

then an explicit expression for  $T$  as a function of  $\bar{A}$  can be obtained;

$$T = \left\{ \frac{K \dot{m} R T_1^\Gamma \bar{A}}{\mu_v [2KL + 2CT_c]^{1/2}} \exp \left( \frac{\mu_v L}{RT_1} \times \left[ 1 + \frac{K}{(1 - K)\alpha} \right] - B \right) \right\}^{1/(\Gamma - 1)} \quad (13)$$

Once  $T$  is determined,  $\theta$ ,  $q$ ,  $u$ ,  $\rho$ , and  $p$  can be computed from Eqs. (5, 4, 3, 2, and 1), respectively.

It is interesting to note that even after the vapor has condensed it is an important factor in determining the way in which the further expansion of the gas occurs. The kinetic and thermal energy of the condensed particles have a large effect on the flow. The assumption sometimes used that the flow far downstream of condensation approximates a frozen isentropic expansion of gas can result in considerable error. (The latter approach means that all condensed particles were somehow removed from the stream without affecting the flow.)

Figure 1 shows a comparison of calculated temperature, pressure, and mass fraction of uncondensed vapor for both an exact (computer) solution and the present approximate solution. The calculations are for a zinc vapor in helium expansion. The stagnation chamber temperature and pressure, and mass fraction of zinc are 4500°K, 4500 psia, and 0.6, respectively. The frozen flow solution is shown for reference. If at each area ratio the latent heat used is adjusted for the change in local temperature, the present solution is very close to the exact solution far downstream of the saturation point as can be seen in the figure. The comparison indicates what might have been expected concerning the region of validity of Eq. (13). The plot of  $q$  at the bottom of the figure shows that by an area ratio of about 15, most of the vapor has been condensed and the present approximate solution is very good downstream of this point.

It can be seen from the figure that the difference in  $q$  between that predicted by the exact solution and by the present approximate one becomes small as  $q$  becomes small. This indicates that a quick check on the region of validity of the approximate solution can be made by calculating  $T$  from Eq. (13) and then computing  $q$  from Eq. (4) to see if  $q$  is indeed small enough for the solution to be valid.

### References

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## Ballistic-Range Measurements of Vibrational-Nonequilibrium Effects for Cone-Cylinders at Supersonic Speeds

GEORGE S. CAMPBELL\* AND PAUL C. KLIMAS†  
University of Connecticut, Storrs, Conn.

### Introduction

THE strong dependence of shock-standoff distance on the degree of vibrational excitation and dissociation in the shock layer has been observed in several published papers. This aerodynamic effect was utilized by Schwartz and Eckerman<sup>1</sup> to determine the vibrational relaxation time of chlorine by firing small spheres in a ballistic range. Aerodynamic calculations for spheres are quite complicated, and so the use of cones with attached shock waves should facilitate comparison between theory and experiment. Ballistic-range experiments by Stephenson<sup>2</sup> were conducted with this in mind. The present studies were undertaken with objectives and experimental methods similar to those of Stephenson. They differ from the NASA studies primarily in the use of a diatomic gas that is much simpler than air and in the emphasis on flow near detachment.

The tests were conducted using 0.375-caliber models fired through a chlorine atmosphere at ambient pressures from 6 to 200 mm Hg. Mach numbers for the tests ranged from approximately 2 to 3, which includes the detachment region for the 45° cone cylinders chosen for the experiments. Many

Received May 1, 1969; revision received July 7, 1969. The authors wish to acknowledge the help of R. L. Lester in carrying out these experiments.

\* Professor of Aerospace Engineering, Associate Fellow AIAA.

† NDEA Fellow; presently at Naval Air Systems Command, Washington, D. C.

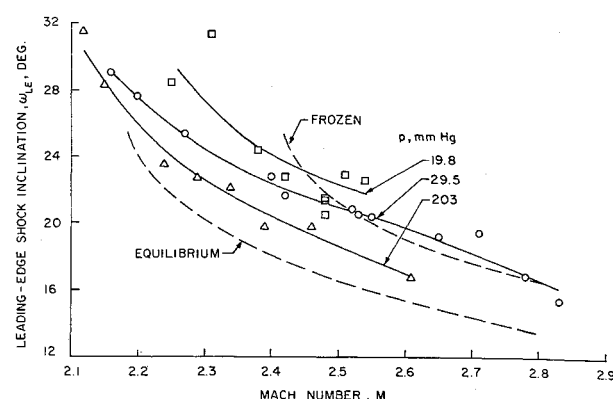


Fig. 1 Leading-edge shock-wave inclination for cone-cylinders in chlorine.

of the details of this investigation, along with numerous theoretical calculations, are given in Ref. 3.

### Measurements and Accuracy

The measurements reported in this Note were obtained by firing the cone-cylinder models through a test section containing chlorine and sealed by thin diaphragms. The cylindrical base of the models had a length to diameter ratio of 1.0, the model diameter being 0.375 in.

Commercially-supplied chlorine (Matheson Company High Purity Grade) was used in the experiments. Water contained in the gas was removed by passing the gas through a moisture absorbent containing phosphorous pentoxide. The maximum impurity level due to residual air in the test section was in all cases under 0.4 mm Hg. Pressure of the test gas was measured by a Texas Instruments Fused Quartz Pressure Gage Model #140 with an open-port Bourdon tube referenced to vacuum.

Model velocity was determined by the breaking of two printed-circuit papers separated by a known distance (12.12 in.  $\pm$  0.03 in.). The total random error of the velocity-measuring system is estimated to be 0.7%, an rms sum of errors associated with the triggering apparatus and the counter used to record the time interval. The second of the velocity-circuit papers triggered a Beckman and Whitley #5205 point light source with a light duration under 1  $\mu$ sec. Other components of the schlieren system used to view the shock wave included two 5-in.-diam lenses, a knife edge, and a projection lens giving a 2:1 magnification at the film plane.

### Results and Discussion

The effect of vibrational nonequilibrium on the shock-wave shape for the cone-cylinder models is shown in Fig. 1, which gives the leading-edge inclination of the shock wave over a range of Mach numbers at several values of ambient pressure. Repeated measurement of the inclination from the schlieren photographs generally resulted in values within 1° or 2° of the mean value plotted in Fig. 1. Also shown in Fig. 1 are equilibrium and frozen-flow predictions for an infinite cone using Dorodnitsyn's one-strip integral technique. The data shown in Fig. 1 exhibit the usual trend of increasing

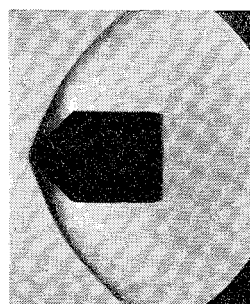


Fig. 2 Schlieren photograph of shock wave about cone-cylinder in chlorine at Mach number of 2.24 and ambient pressure of 202 mm Hg.